Performance Analysis of output SNR over Weibull-Gamma Fading Channel

Amandeep Kaur[1], *Jyoteesh Malhotra*[2]
Department of Mathematics[1], *ECE Department*[2]
Guru Nanak Dev. University College[1] *, Guru Nanak Dev University Regional Campus*[2]
hargobind.ji@gmail.com[1] *, jyoteesh@gmail.com*[2]

Abstract : Weibull-Gamma heterogeneous channel model is versatile, flexible and multiparameter channel model. Weibull which is terrain specific channel model is selected along with gamma channel model. Expressions of higher order moments are obtained which are used to evaluate the effect of skewness and kurtosis of the output signal to noise ratio(SNR). Considering the multi parameter model, different situations of skewness and kurtosis under various fading conditions are studied to characterize the statistical behavior of the output SNR .

Keywords:- Weibull-Gamma ; Skewness ; Kurtosis ; Mellin transformation ; Fading channel , Heterogeneous channel

INTRODUCTION

Due to increasing demand of wireless based services, the wireless systems are required to operate in increasingly hostile environments. In real time propagation scenario the signal leaving the transmitter reaches the receiver after multiple scattering and both multipath fading and shadowing occur simultaneously .Such situations where both multipath fading and shadowing occurs simultaneously are characterized by heterogeneous environment instead of homogeneous environment. Literature [Shankar P.M.,(2012)] is full of various fading channel models according to the need of various propagation environments. Various compound lognormal channel models are also available. But these compound lognormal channel models involve complex and complicated mathematical calculations to solve the lognormal expression. Vast research is already available for the performance analysis of Weibull channel model and gamma channel model separately. However, little attention has been paid to the heterogeneous weibull-gamma composite channel model. Heterogeneous Weibull-Gamma channel model is a mixture of Weibull and gamma model which does not involve any complex lognormal expression. This model has been in use in recent past. In [T. Reddy *et al*, (2007)] outage probability of exponentially correlated weibull-gamma distribution was studied. In [Nadarajah, and Kotz S, (2007)] and [Bithas ,(2009)] Probability density function (PDF) and Characteristic function (CF) of weibull-gamma channel model has been obtained. However analysis of higher order moments of instantaneous SNR for weibull-gamma channel model has never been done before. Higher order moments are useful in signal processing algorithms for signal detection, classification, and estimation since they play an important role for the analysis of the performance of wideband communications systems in the presence of fading. Skewness and kurtosis of the output SNR of correlated Nakagami-m fading channels are studied in [Karagiannidis G. K. *et al*,(2004)].In [M. Z. Win,(2003)] skewness and kurtosis for various antenna subset diversity schemes are derived. The concept of skewness has been used in [Hao Zhang *et al*, (2012)] to improve the precision of the time of arrival estimation.

In this paper attempt has been made using Mellin transformation technique to evaluate the higher order moments of heterogeneous weibull-gamma channel model. Higher order moment based metrics, such as Skewness which is a measure of the symmetry and Kurtosis which is defined as the degree of peakedness in the distribution are evaluated to characterize the distribution of output SNR.

The rest of the paper is organized as follows. Section 1 contains system and channel model. Higher order moments based measures i.e. Skewness and Kurtosis of transmitted and output SNR is discussed in section 2 before the paper is finally concluded in section 3.

1. SYSTEM AND CHANNEL MODEL

Consider the real-time heterogeneous environment where the signal is transmitted over Weibull-gamma composite fading channel model .

Let ‘X’ and ‘Y’ be the random variables distributed over Weibull channel and Gamma channel respectively.

The pdf of Weibull model is given as :

$$f_X(x) = \frac{cx^{c-1}}{\lambda^c}\exp(-\frac{x^c}{\lambda^c}) \qquad (1)$$

where “c” and “λ” represents the fading and shaping parameter.

The pdf of gamma model is given as :

$$f_Y(y) = \frac{y^{m-1}}{\beta^m\Gamma(\mathrm{m})}\exp(-\frac{y}{\beta}) \qquad (2)$$

where m is the fading parameter and β is scaling factor.

Using Mellin transformation method, moments of (1) and (2) are evaluated. The Mellin transform of any function $p_X(x)$ is given as ([Debnath L. and Bhatta D.,(2007)], eq. (8.2.5))

$$\varphi_X(p(x),s) = E(X^{s-1}) = \int_0^\infty p(x)x^{s-1}\,dx$$

Using the above formula the Mellin transformation of PDF of Weibull $f_X(x)$ and Gamma channel model “$f_Y(y)$”are given as:

$$\varphi_X(\mathrm{s}) = \Gamma(1+\frac{s-1}{c})\lambda^{S-1}$$

$$\varphi_Y(\mathrm{s}) = \frac{\Gamma(\mathrm{m}+s-1)}{\Gamma(\mathrm{m})}\beta^{S-1}$$

Let “R” represents the random variable of product Weibull-Gamma model. Using product convolution property of Mellin transform, the Mellin transformation of compound heterogeneous Weibull-Gamma channel model is

$$\varphi_R(\mathrm{s}) = \frac{(\lambda\beta)^{S-1}}{\Gamma(\mathrm{m})}\Gamma(\mathrm{m}+\mathrm{s}-1)\Gamma(1+\frac{s-1}{c}) \qquad (3)$$

The nth order moment of Weibull-Gamma is obtained by replacing s-1 by n in Eq.(3)

$$\mu_n^{'} = E(\mathrm{R}^n) = (\lambda\beta)^n\frac{\Gamma(\mathrm{m}+n)\Gamma(1+\frac{n}{c})}{\Gamma(\mathrm{m})} \qquad (4)$$

The received SNR at the output is given as

$$\gamma = \frac{E_s}{N_o}R^2$$

where E_s and N_0 are resp. the transmitted signal’s average energy and one sided additive white Gaussian noise.

The average SNR at the output (destination) is

$$\bar{\gamma} = \frac{E_s}{N_o}E(\mathrm{R}^2)$$

Mellin transformation of output SNR is given as

$$\varphi_\gamma(\mathrm{s}) = \mathrm{E}(\gamma^{s-1})$$

$$\varphi_\gamma(\mathrm{s}) = \left(\frac{E_s}{N_0}\right)^{s-1}\varphi_R(2\mathrm{s}-1)$$

Replacing s by 2s-1 in (3), the nth order moment of output SNR (γ) is given as

$$v_n^{'} = E(\gamma^n) = (\frac{E_s}{N_0})^n\frac{(\lambda\beta)^{2n}}{\Gamma(\mathrm{m})}\Gamma(\mathrm{m}+2n))\Gamma(1+\frac{2n}{c}) \qquad (5)$$

From equations (4) and (5) various higher order moment based performance measures can be evaluated .

2. PERFORMANCE METRICS

In order to make comparison between two or more channel models we study mainly three characteristics namely mean, skewness and kurtosis. These measures are based on the concept of higher order moments.. Skewness and kurtosis are two measures which study the deviation or scatterings of signal from the mean value.

2.1 Skewness

It is the measure of degree of asymmetry for the distribution of transmitted or received signal . It is helpful in making comparative study. This measure is based on the knowledge of second and third order moments. More the skewness , more is the scatterings or variability of signal about the mean value and hence lesser will be the stability. The coefficient of skewness is denoted by β_1.

For $\beta_1 = 0$ (distribution curve is normal)

$\beta_1 < 0$ (distribution curve is negatively skewed i.e. scatterness of data is more towards left)

$\beta_1 > 0$ (distribution curve is positively skewed i.e. scatterness of data is more towards right)

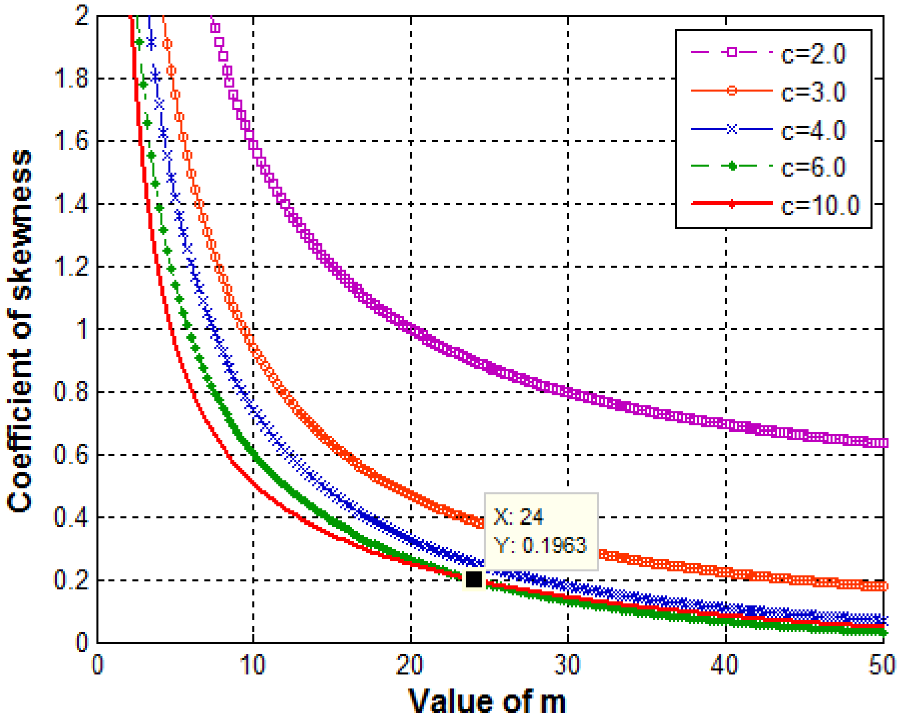
Coefficient of skewness of transmitted signal is:

$$\frac{(\mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3)^2}{(\mu_2' - (\mu_1')^2)^3}$$

Coefficient of skewness of output SNR is:

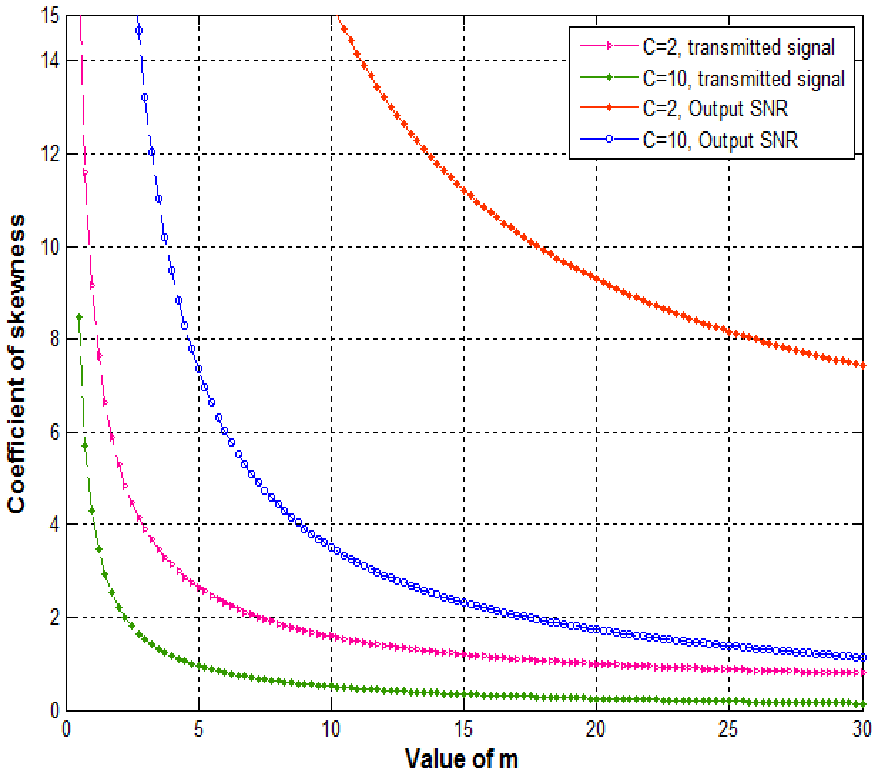
$$\frac{(\nu_3' - 3\nu_1'\nu_2' + 2(\nu_1')^3)^2}{(\nu_2' - (\nu_1')^2)^3}$$

This measure helps in studying the variability of signal with the variation of fading parameter. Coefficient of skewness of transmitted signal with the variation of fading parameter 'm' is given in Fig (1).



Fig(1). Coefficient of skewness of the transmitted signal

The distribution or transmitted signal is positively skewed as the value of coefficient of skewness is greater than zero. Overall skewness decreases with increase in value of m. There is abrupt decrease in the value of skewness from c=2 to c=3 but after that skewness decreases slowly and gradually. The value of coefficient of skewness is same at m=24, c=6.0 and m=24,c=10.0 and after that decrease in skewness is more for c=6.0 than for c=10.0 which shows that effect of Weibull channel increases than gamma channel. Comparison of coefficient of skewness of output SNR with transmitted signal is done in Fig(2).



Fig(2). Coefficient of skewness of transmitted signal and output SNR with the variation of fading parameter m

Coefficient of skewness of output SNR is more than that of transmitted signal which indicates that the scattering of output SNR is more than the transmitted signal but decreases with increase in value of c.

2.2 Kurtosis

It is the degree of peakedness or bulginess of the transmitted signal or received SNR around the mean value. This measure is based on fourth order moment .Unlike skewness which consider the scattering of signal towards left or right of the normal position , kurtosis considers the concentration of signal near the mean value i.e. upward or downward from the normal position. The coefficient of kurtosis is defined by β_2 .

For $\beta_2 = 3$ (distribution curve is normal / Mesokurtic)

$\beta_2 < 3$ (distribution curve is platykurtic i.e. peak of curve is less than normal curve)

$\beta_2 > 3$ (distribution curve is leptokurtic i.e. peak of curve is more than normal curve)

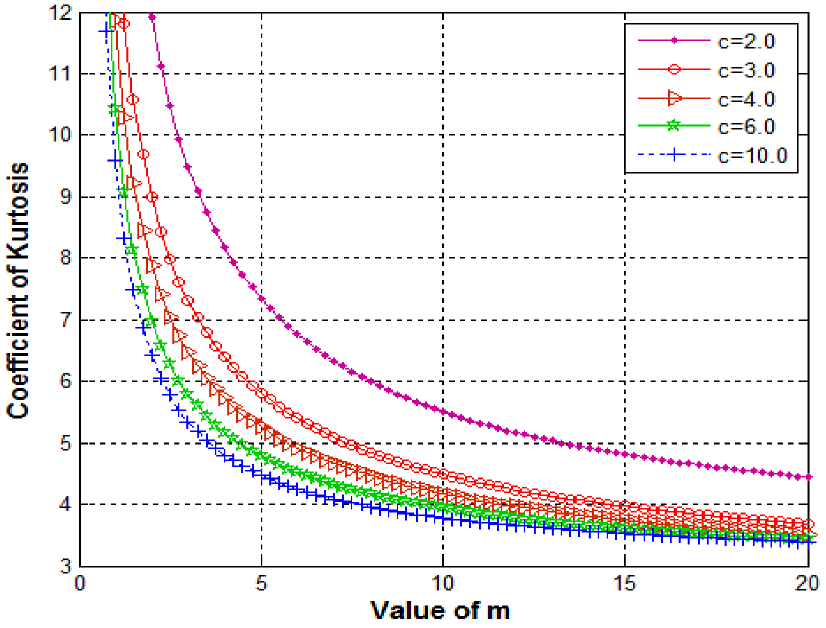
Coefficient of kurtosis of transmitted signal is

$$\frac{\mu_4' - 4\mu_1'\mu_3' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4}{(\mu_2' - (\mu_1')^2)^2}$$

Coefficient of kurtosis of output SNR

$$\frac{\nu_4' - 4\nu_1'\nu_3' + 6\nu_2'(\nu_1')^2 - 3(\nu_1')^4}{(\nu_2' - (\nu_1')^2)^2}$$

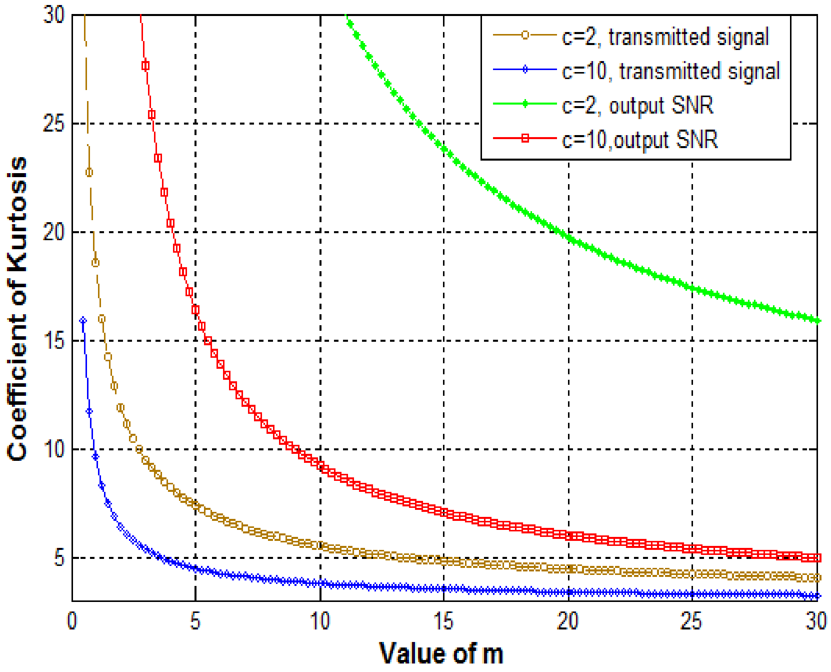
Coefficient of kurtosis of transmitted signal with the variation of fading parameter ‘m’ is given in Fig(3).



Fig(3). Coefficient of kurtosis of transmitted signal

The value of coefficient of kurtosis in all cases is greater than 3 , which shows that the distribution is leptokurtic .But as the value of c increases the graph tends to approach normal distribution i.e. Mesokurtic .

Further for large values of m there is minor change in value of coefficient of kurtosis for some fixed value of c. Comparison of coefficient of kurtosis of output SNR with transmitted signal is done in Fig(4).



Fig(4). Coefficient of kurtosis of transmitted signal and output SNR with the variation of fading parameter m

From graph it is clear that coefficient of kurtosis of output SNR is greater than that of the transmitted signal. Also coefficient of kurtosis decreases with increase in value of c and with increase in value of m. Even the difference in the value of coefficient of kurtosis for output SNR and transmitted signal decreases with increase in value of c. For sufficiently large values of m i.e. for m→∞ the value of coefficient of kurtosis remains almost constant for some fixed value of c. From graph it is clear that the difference between the values of skewness between output SNR and transmitted signal at m=20 for c=2 is more than for c=10.

3. CONCLUSION

Mellin transformation technique has been used here to derive higher order statistics of the output SNR of Weibull-Gamma channel model in a multipath fading environment. Higher order moment based performance measures such as skewness and kurtosis were evaluated. Effect of variation of Skewness and Kurtosis of both transmitted signal and output SNR has been studied. Based on skewness and kurtosis peculiar behavior of Fading Channel model was analyzed for different fading situations by varying the values of fading parameters. The novel results obtained here can be useful for wireless system designers in studying the stability of system through diverse compound fading situations created using Weibull-Gamma model.

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